Reinvestigation of Inhomogeneous Superconductors and High T_c Superconductors

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Abstract

We review the recent progress in the theory of inhomogeneous superconductors. It was shown that Gor'kov's self-consistency equation needs a pairing constraint derived from the Anomalous Green's function. The Bogoliubov-de Gennes equations also need a pairing constraint in order to obtain a correct vacuum state by the corresponding unitary transformation. This new study opens up a reinvestigation of inhomogeneous superconductors. We discuss (i) problems of the conventional Green's function theory, (ii) reinvestigation of impure superconductors, and (iii) impurity doping effect in high T_c superconductors. It is also pointed out that a new formalism is required to tackle the macroscopically or mesoscopically inhomogeneous systems such as the junction and the vortex problems.

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Inhomogeneous Superconductors

and

High T_c Superconductors

$$\Delta(\mathbf{r}) = VT\sum_{\omega}\int\{G^{\uparrow}_{\omega}(\mathbf{r},\mathbf{l})G^{\downarrow}_{-\omega}(\mathbf{r},\mathbf{l})\}^{\mathrm{P}}\Delta(\mathbf{l})\mathrm{d}\mathbf{l} \qquad (1)$$

P : Pairing constraint

Yong-Jihn Kim

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Contents

Chapter 1 Introduction

1. Problems of the conventional Green's function theory

1.1. Remarks on the failure of Green's function theory

Impure superconductors

• "Theory of superconductivity", by *P. W. Anderson*, rapporteur's remarks at Toronto LTIV, 1960.

By the use of Gor'kov techniques, Abrikosov and Gor'kov have succeeded in obtaining a perturbation theory valid for very small energy gap $\cdots$, it is entirely <u>incorrect</u> as far as any physical results are concerned.

• "Breakdown of Eliashberg theory for two-dimensional superconductivity in the presence of disorder" by *R. C. Dynes, A. E. White, J. M. Graybeal, and J. P. Garno*, Phys. Rev. Lett. **57**, 2195 (1986).

Abstract: We conclude that the standard picture of enhanced Coulomb repulsion

with increasing sheet resistance is too naive and that, in the presence of disorder, an analysis more sophisticated than Eliashberg theory is necessary.

- "Apparent destruction of superconductivity in the disordered one-dimensional limit" by *J. M. Graybeal, P. M. Mankiewich, R. C. Dynes, and M. R. Beasley*, Phys. Rev. Lett. **59**, 2697 (1987).

Abstract: Our findings are in clear dsagreement with a recent theoretical treatment.

- "Superconducting-Insulating transition in two-dimensional a-MoGe thin films by *A. Yazdani and A. Kapitulnik*, Phys. Rev. Lett. **74**, 3037 (1995).

Abstract: However, contrary to the theoretical predictions we find the critical resistance to be sample dependent.

Junction Problems

- "Boundary-condition effects on the superconducting transition temperature of proximity-effect systems", by *P. R. Broussard*, Phys. Rev. B **43**, 2783 (1991).

Abstract: The superconucting critical temperature, T_c, of different configurations of layers is studied under the de Gennes-Werthamer model. Certain Inconsistencies are seen to develop with the use of this approach, calling into question previous results obtained.

- "Introduction to superconductivity", by *M. Tinkham*, McGraw-Hill, 1975.

P. 195.

Thus, for $V \neq 0$, and for an arbitray type of weak-link element, we may generalize (6-4) to

$$I = I_o sin\gamma + (G_o + G_{int} cos\gamma)V \qquad (6\text{-}4a)$$

where G_o, G_{int} themselves may be functions of V. Experiments of Pedersen et al.[1] on tunnel junctions, of Falco et al.[2] on thin-film weak links, and of Vincent and Deaver[3]

on point-contact weak links have all demonstrated the existence of this pair-quasi-particle interference term, and all have shown that $G_{int}/G_o \approx \underline{-1}$. This result has drawn much recent attention, because it appears that the microscopic theory[4] yields a <u>positive sign</u> for this ratio.

- "Superconducting weak links", by *K. K. Likharev*, Rev. Mod. Phys. **51**, 101 (1979).

B. Unsolved problems

2. ac processes

(1) Is the discrepancy in the signs of the same two terms between experiment and the Mitsai theory indicative of certain <u>fundamental inconsistencies</u> in the Green's function formulation of the theory of nonstationary superconductivity?

C. Concluding remarks

$\cdots$ in the field of ac effects there are more questions than answers.

1.2. Discrepancy between Pippard theory and Ginzburg-Landau theory

Pippard-BCS theory

Pippard assumed that the current $\mathbf{j}(\mathbf{r})$ at one point will depend on the vector potential $\mathbf{A}(\mathbf{r}')$ at all neighboring points $\mathbf{r}'$ such that $|\mathbf{r}-\mathbf{r}'| < \xi_o$. Here ξ_o means the BCS coherence length

$$\xi_o = \frac{\hbar v_F}{\pi \Delta}. \tag{2}$$

The Pippard nonlocal current relation becomes

$$\mathbf{j}(\mathbf{r}) = -\frac{3}{4\pi \xi_o c \Lambda} \int \frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{E}(\mathbf{r}')]}{R^4} e^{-R/\xi_o} d^3 r', \qquad \mathbf{R} = \mathbf{r} - \mathbf{r}'. \tag{3}$$

Note that the range of nonlocality is reduced by a factor of about 0.75 on going from $T = 0$ to T_c.

It is a tribute to Pippard's insight into the physics of superconductivity that his equation is almost identical to that given by BCS theory.

Ginzburg-Landau theory

The fundamental Landau-Ginzburg equations are the following:

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - \frac{2e\mathbf{A}}{c})^2\psi = 0, \tag{4}$$

$$\mathbf{j} = \frac{e\hbar}{im}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{4e^2}{mc}\psi^*\psi\mathbf{A}. \tag{5}$$

Even though the relation between $\mathbf{j}$ and $\mathbf{A}$ is approximated by a local relation, the Ginzburg-Landau theory definitely includes nonlocal effects and the coherence length appears in a natural way. The Ginzburg-Landau coherence length ξ_{GL} is given by

$$\xi_{GL} = 0.74\frac{\xi_o}{\sqrt{1 - T/T_c}}. \tag{6}$$

(From J. R. Schrieffer, Theory of Superconductivity, p. 22, Benjamin, 1964.)

However, the Ginzburg-Landau coherence length ξ_{GL} is very different from the BCS coherence length ξ_o. (From Fetter and Walecka, Quantum theory of many particle systems, p. 433, 1971.)

Note that the range of nonlocality in the Pippard-BCS theory is reduced by a factor of about 0.75 on going from $T = 0$ to T_c. Since the Pippard-BCS theory does not allow the spatial variation of the energy gap or the order parameter $\psi(\mathbf{r})$, the nonlocal effect in the Pippard-BCS theory is different from that in the Ginzburg-Landau theory. In other words, we should compare the Pippard nonlocal relation Eq. (3) with the Ginzburg-Landau local relation Eq. (5) for a spatially uniform order parameter $\psi(\mathbf{r})$.

In conclusion, the Ginzburg-Landau theory is in serious conflict with the Pippard-BCS theory.

1.3. Correspondence principle and theory of superconductivity

1.3.1. Abstract for 1998 Theoretical Solid State Physics Symposium

Taejon, Feb. 19-21, 1998, Korea

Correspondence Principle and Theory of Superconductivity†

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Abstract

The conventional theory of superconductivity has a hierarchical structure. After the microscopic theory by Bardeen, Cooper, and Schrieffer, there appeared Gor'kov formalism, the Bogoliubov-de Gennes equations, and the Eliashberg formalism which tried to generalize the BCS theory. On the macroscopic level, the London theory, the Pippard's nonlocal theory, and the Ginzurg-Landau theory form such a hierarchical structure. We discuss the correspondence between the theories. It is shown that the correspondence principle leads to the microscopic pairing constraint and therefore the theory of inhomogeneous superconductors needs to be reinvestigated. In particular, since the Ginzburg-Landau theory lacks the information of the pairing correlations it gives rise to different nonlocal electrodynamics from those of the Pippard and the BCS theories.

† To the memory of Professor D. J. Kim.

1.3.2. Correspondence between BCS theory and Gor'kov formalism

BCS Theory

For a homogeneous system,

$$H_{red} = \sum_{\vec{k}\vec{k}'} V_{\vec{k}\vec{k}'} c^{\dagger}_{\vec{k}'} c^{\dagger}_{-\vec{k}'} c_{-\vec{k}} c_{\vec{k}}, \tag{7}$$

where

$$V_{\vec{k}\vec{k}'} = \begin{cases} -V, & \text{if } |\epsilon_{\vec{k}}|, |\epsilon_{\vec{k}'}| \leq \omega_D \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

This reduction procedure is recognizing in advance which eigenstates will be paired and so contribute to the BCS condensate.

The BCS gap equation:

$$\Delta_{\vec{k}} = \sum_{n'} V_{\vec{k}\vec{k}'} \frac{\Delta_{\vec{k}'}}{2E_{\vec{k}'}} tanh \frac{E_{\vec{k}'}}{2T}. \tag{9}$$

Gor'kov formalism

In Gor'kov formalism, a point interaction $-V\delta(\mathbf{r}_1 - \mathbf{r}_2)$ is used for the pairing interaction between electrons. For a homogeneous system, the pairing interaction is

$$\begin{aligned} H_G &= -\frac{1}{2}V \int d\mathbf{r} \sum_{\alpha\beta} \Psi^\dagger(\mathbf{r}\alpha)\Psi^\dagger(\mathbf{r}\beta)\Psi(\mathbf{r}\beta)\Psi(\mathbf{r}\alpha) \\ &= -\frac{1}{2}V \sum_{\vec{k}\vec{k}'\vec{q}\sigma\sigma'} c^\dagger_{\vec{k}-\vec{q},\sigma} c^\dagger_{\vec{k}'+\vec{q},\sigma'} c_{\vec{k}'\sigma'} c_{\vec{k},\sigma}, \end{aligned} \tag{10}$$

and

$$\begin{aligned} V_{\vec{k}\vec{k}'} &= -V \int \phi^*_{\vec{k}'}(\mathbf{r})\phi^*_{-\vec{k}'}(\mathbf{r})\phi_{-\vec{k}}(\mathbf{r})\phi_{\vec{k}}(\mathbf{r})d\mathbf{r} \\ &= -V. \end{aligned} \tag{11}$$

The self-consistency equation is

$$\Delta(\mathbf{r}) = VT \sum_{\omega} \int \Delta(\mathbf{l}) G^{\uparrow}_{\omega}(\mathbf{r}, \mathbf{l}) G^{\downarrow}_{-\omega}(\mathbf{r}, \mathbf{l}) d\mathbf{l}. \tag{12}$$

Correspondence principle

Note that the two points are not clear in Gor'kov's formalism, i.e., the BCS reduction procedure and the retardation cutoff. To obtain the same result as that of the BCS theory, these two ingredients should be taken care of in some way. As will be shown later, the negligence of the BCS reduction procedure causes a serious pairing problem especially in impure superconductors.

The correspondence principle leads to the revised self-consistency equation,

$$\Delta(\mathbf{r}) = VT\sum_{\omega}\int \Delta(\mathbf{l})\{G_{\omega}^{\uparrow}(\mathbf{r},\mathbf{l})G_{-\omega}^{\downarrow}(\mathbf{r},\mathbf{l})\}^{P}d\mathbf{l}, \tag{13}$$

where P denotes a pairing constraint, which is derived from the physical constrant of the Anomalous Green's function,

$$F(\mathbf{r},\mathbf{r}') = F(\mathbf{r}-\mathbf{r}'). \tag{14}$$

Notice that Eq. (13) is nothing but another form of the BCS gap equation.

1.3.3. Correspondence between BCS theory and Eliashberg formalism

Eliashberg theory with pairing constraint

The conventional self-consistency equation for the pair potential is

$$\begin{aligned}&\Delta^{*}(\omega_n,\mathbf{r})Z(\omega_n)\\ &\quad = \gamma^2 T\sum_{n'}\lambda(\omega_n,\omega_{n'})\int d\mathbf{r}_o G_N^{\uparrow}(-\omega_{n'},\mathbf{r}_o,\mathbf{r})G_N^{\downarrow}(\omega_{n'},\mathbf{r}_o,\mathbf{r})\Delta^{*}(\omega_{n'},\mathbf{r}_o)Z(\omega_{n'}).\end{aligned} \tag{15}$$

From the physical constraint of the Anomalous Green's function, i.e.,

$$F^{+}(\omega_n,\mathbf{r},\mathbf{r}') = F^{+}(\omega_n,\mathbf{r}-\mathbf{r}'), \tag{16}$$

we find the revised self-consistency equation,

$$\begin{aligned}&\Delta^{*}(\omega_n,\mathbf{r})Z(\omega_n) =\\ &\quad \gamma^2 T\sum_{n'}\lambda(\omega_n,\omega_{n'})\int d\mathbf{r}_o\{G_N^{\uparrow}(-\omega_{n'},\mathbf{r}_o,\mathbf{r})G_N^{\downarrow}(\omega_{n'},\mathbf{r}_o,\mathbf{r})\}^{P}\Delta^{*}(\omega_{n'},\mathbf{r}_o)Z(\omega_{n'}).\end{aligned} \tag{17}$$

Since

$$\Delta^{*}(\omega_n,m) = \int \psi_m(\mathbf{r})\psi_m^{*}(\mathbf{r})\Delta^{*}(\omega_n,\mathbf{r})d\mathbf{r}, \tag{18}$$

the strong-coupling gap equation is

$$\Delta^{*}(\omega_n,m)Z(\omega_n) = T\sum_{n'}\lambda(\omega_n,\omega_{n'})\sum_{m'}V_{mm'}\frac{\Delta^{*}(\omega_{n'},m')Z(\omega_{n'})}{[\omega_{n'}Z(\omega_{n'})]^2+\epsilon_{m'}^2}, \tag{19}$$

where

$$V_{mm'} = \gamma^2\int |\psi_m(\mathbf{r})|^2|\psi_{m'}(\mathbf{r})|^2 d\mathbf{r}. \tag{20}$$

Weak-coupling limit

The strong-coupling theory leads to the weak-coupling theory in the static limit, (i.e.),

$$\Delta^*(\omega_n, \mathbf{r}) = \Delta^*(0, \mathbf{r}) = \Delta^*(\mathbf{r}), \tag{21}$$

$$Z(\omega) = Z(0) = 1, \tag{22}$$

$$\lambda(\omega_n, \omega_{n'}) = \lambda(0, 0) = 1. \tag{23}$$

Accordingly, we find

$$\Delta^*(\mathbf{r}) = \gamma^2 T \sum_{n'} \int d\mathbf{r}_o G_N^{\uparrow}(-\omega_{n'}, \mathbf{r}_o, \mathbf{r}) G_N^{\downarrow}(\omega_{n'}, \mathbf{r}_o, \mathbf{r}) \Delta^*(\mathbf{r}_o), \tag{24}$$

$$\Delta^*(\mathbf{r}) = \gamma^2 T \sum_{n'} \int d\mathbf{r}_o \{G_N^{\uparrow}(-\omega_{n'}, \mathbf{r}_o, \mathbf{r}) G_N^{\downarrow}(\omega_{n'}, \mathbf{r}_o, \mathbf{r})\}^P \Delta^*(\mathbf{r}_o), \tag{25}$$

and

$$\begin{aligned} \Delta^*(m) &= T \sum_{n'} \sum_{m'} V_{mm'} \frac{\Delta^*(m')}{\omega_{n'}^2 + \epsilon_{m'}^2} \\ &= \sum_{m'} V_{mm'} \frac{\Delta^*(m')}{2\epsilon_{m'}} tanh(\frac{\epsilon_{m'}}{2T}). \end{aligned} \tag{26}$$

Correspondence

Note that the correspondence principle, which relates strong-coupling and weak-coupling theories, works only when the pairing constraint is incorporated into the self-consistency equation. For more details, see Y.-J. Kim, "Strong-coupling theory of impure superconductors: Correspondence with weak-coupling theory".

1.3.4. Correspondence between Pippard-BCS theory and Ginzburg-Landau theory

Superconductivity is a typical example of macroscopic quantum phenomena which can be explained only by the quantum mechanics. Since the Ginzburg-Landau theory was constructed in 1950 before the pairing theory of BCS, it lacks the microscopic information of the pairing correlation and leads to different nonlocal electrodynamics from that of the Pippard-BCS theory. (See also Ch. 1.2.) Gor'kov's microscopic

derivation of the Ginzburg-Landau theory is not valid because he used the incorrect self-consistency equation.

We should generalize the Ginzburg-Landau theory by including the information of the pairing correlation.

1.4. Pairing constraint on Gor'kov formalism

From Y.-J. Kim, "A constraint on the Anomalous Green'sfunction", Mod. Phys. Lett. B, (1996).

Homogeneous System

The conventional self-consistency equation is

$$\Delta(\mathbf{r}) = VT\sum_{\omega}\int \Delta(\mathbf{l})G^{\uparrow}_{\omega}(\mathbf{r},\mathbf{l})G^{\downarrow}_{-\omega}(\mathbf{r},\mathbf{l})d\mathbf{l}. \tag{27}$$

From the physical constraint of the Anomalous Green's function,

$$F(\mathbf{r},\mathbf{r}') = F(\mathbf{r}-\mathbf{r}'), \tag{28}$$

we obtain the revised self-consistency equation,

$$\Delta(\mathbf{r}) = VT\sum_{\omega}\int \Delta(\mathbf{l})\{G^{\uparrow}_{\omega}(\mathbf{r},\mathbf{l})G^{\downarrow}_{-\omega}(\mathbf{r},\mathbf{l})\}^{P}d\mathbf{l}, \tag{29}$$

where P denotes a pairing constraint which dictates pairing between $\vec{k}\uparrow$ and $\vec{k}\downarrow$.

Accordingly, the revised strong-coupling self-consistency equation is

$$\Delta^{*}(\omega_n,\mathbf{r})Z(\omega_n) = \gamma^2 T\sum_{n'}\lambda(\omega_n,\omega_{n'})\int d\mathbf{r}_o\{G^{\uparrow}_{N}(-\omega_{n'},\mathbf{r}_o,\mathbf{r})G^{\downarrow}_{N}(\omega_{n'},\mathbf{r}_o,\mathbf{r})\}^{P}\Delta^{*}(\omega_{n'},\mathbf{r}_o)Z(\omega_{n'}). \tag{30}$$

Dirty System

From the physical constraint of the Anomalous Green's function, i.e.,

$$\overline{F(\mathbf{r},\mathbf{r}',\omega)}^{imp} = \overline{F(\mathbf{r}-\mathbf{r}',\omega)}^{imp}, \tag{31}$$

the revised self-consistency equation is

$$\Delta(\mathbf{r}) = VT\sum_{\omega}\int \Delta(\mathbf{l})\{G^{\uparrow}_{\omega}(\mathbf{r},\mathbf{l})G^{\downarrow}_{-\omega}(\mathbf{r},\mathbf{l})\}^{P}d\mathbf{l}, \tag{32}$$

where P denotes the Anderson's pairing condition. Notice that Eq. (32) is nothing but another form of the BCS gap equation,

$$\Delta_n = \sum_{n'} V_{nn'}\frac{\Delta_{n'}}{2E_{n'}}tanh\frac{E_{n'}}{2T}. \tag{33}$$

1.5. Pairing constraint on the Bogoliubov-de Gennes equations

From Y.-J. Kim, "Pairing in the Bogoliubov-de Gennes equations", Int. J. Mod. Phys. Lett. B, (1997).

1.5.1. Dirty System

In 1959, P. W. Anderson introduced a ground state for dirty superconductors, which is given by

$$\tilde{\phi}_{Anderson} = \prod_{n}(u_n + v_n c^{\dagger}_{n\uparrow}c^{\dagger}_{\bar{n}\downarrow})|0>, \tag{34}$$

where $c^{\dagger}_{\bar{n}\downarrow}$ is the creation operator for an electron in the scattered-state $\psi^*_n(\mathbf{r})|\downarrow>$. On the other hand, it has been claimed that the energy is lowered if we pair states Φ_n which are better choices than ψ_n by using the Bogoliubov-de Gennes equations. The state Φ_n is basically a linear combination of the normal scattered states. The coupling comes from the pair potential. However, we show that pairing $\Phi_n\uparrow$ and $\Phi_{\bar{n}}\downarrow$ leads to the violation of the physical constraint of the system.

The unitary transformation,

$$\begin{aligned}\Psi(\mathbf{r}\uparrow) &= \sum_n(\gamma_{n\uparrow}u_n(\mathbf{r}) - \gamma^{\dagger}_{n\downarrow}v^*_n(\mathbf{r})),\\ \Psi(\mathbf{r}\downarrow) &= \sum_n(\gamma_{n\downarrow}u_n(\mathbf{r}) + \gamma^{\dagger}_{n\uparrow}v^*_n(\mathbf{r})),\end{aligned} \tag{35}$$

leads to the following Bogoliubov-de Gennes equations,

$$\begin{aligned}\epsilon u(\mathbf{r}) &= [H_e + U(\mathbf{r})]u(\mathbf{r}) + \Delta(\mathbf{r})v(\mathbf{r}),\\ \epsilon v(\mathbf{r}) &= -[H^*_e + U(\mathbf{r})]v(\mathbf{r}) + \Delta^*(\mathbf{r})u(\mathbf{r}).\end{aligned} \tag{36}$$

To find the vacuum state for γ particles, we expand the field operator by the scattered states:

$$\Psi(\mathbf{r}\alpha) = \sum_n \psi_n(\mathbf{r}) c_{n\alpha}. \tag{37}$$

Then it can be shown

$$\begin{aligned} \gamma_{n\uparrow} &= \sum_{n'} (u^*_{n,n'} c_{n'\uparrow} + v_{n,n'} c^\dagger_{n'\downarrow}) = U_n b_{n\uparrow} - V_n b^\dagger_{\bar{n}\downarrow}, \\ \gamma_{n\downarrow} &= \sum_{n'} (u^*_{n,n'} c_{n'\downarrow} - v_{n,n'} c^\dagger_{n'\uparrow}) = U_n b_{n\downarrow} + V_n b^\dagger_{\bar{n}\uparrow}, \end{aligned} \tag{38}$$

where

$$\begin{aligned} u_{n,n'} &= \int \psi^*_{n'}(\mathbf{r}) u_n(\mathbf{r}) d\mathbf{r}, \\ v_{n,n'} &= \int \psi^*_{n'}(\mathbf{r}) v^*_n(\mathbf{r}) d\mathbf{r}. \end{aligned} \tag{39}$$

Finally, we obtain

$$\tilde{\phi}_{BdG} = \prod_n (U_n + V_n b^\dagger_{n\uparrow} b^\dagger_{\bar{n}\downarrow})|0> . \tag{40}$$

Note that the Bogoliubov-de Gennes equations, Eq.(36) correspond to the vacuum state where $\tilde{\Phi}_n(\mathbf{r})[= \frac{1}{U_n} u_n(\mathbf{r})] \uparrow$ and $\tilde{\Phi}_{\bar{n}}(\mathbf{r})[= -\frac{1}{V_n} v^*_n(\mathbf{r})] \downarrow$ (instead of $\psi_n(\mathbf{r}) \uparrow$ and $\psi_{\bar{n}}(\mathbf{r}) \downarrow$) are paired.

Now we must decide which is the correct ground state in the presence of impurities. Above all, the correct ground state should satisfy the physical constraint of the system. It can be shown that the state $\tilde{\phi}_{BdG}$ gives the 'averaged' pair potential

$$\overline{\Delta(\mathbf{r})}^{imp} \neq \text{constant}, \tag{41}$$

which violates the physical constraint of the system. Therefore the correct ground state is $\tilde{\Phi}_{Anderson}$. To obtain $\tilde{\Phi}_{Anderson}$ from the Bogoliubov-de Gennes equations, we need a pairing constraint:

$$\tilde{\phi}_{BdG} = \tilde{\phi}_{Anderson}, \tag{42}$$

which gives

$$u_n(\mathbf{r}) \propto \psi_n(\mathbf{r}),$$
$$v_n^*(\mathbf{r}) \propto \psi_{\bar{n}}(\mathbf{r}). \tag{43}$$

1.5.2. Homogeneous System

The unitary transformation,

$$\Psi(\mathbf{r}\uparrow) = \sum_n (\gamma_{n\uparrow} u_n(\mathbf{r}) - \gamma_{n\downarrow}^\dagger v_n^*(\mathbf{r})),$$
$$\Psi(\mathbf{r}\downarrow) = \sum_n (\gamma_{n\downarrow} u_n(\mathbf{r}) + \gamma_{n\uparrow}^\dagger v_n^*(\mathbf{r})), \tag{44}$$

leads to

$$\gamma_{n'\uparrow} = \sum_{\vec{k}} (u_{n',\vec{k}}^* a_{\vec{k}\uparrow} + v_{n',\vec{k}} a_{\vec{k}\downarrow}^\dagger) = U_n a_{n\uparrow} - V_n a_{\bar{n}\downarrow}^\dagger,$$
$$\gamma_{n'\downarrow} = \sum_{\vec{k}} (u_{n',\vec{k}}^* a_{\vec{k}\downarrow} - v_{n',\vec{k}} a_{\vec{k}\uparrow}^\dagger) = U_n a_{n\downarrow} + V_n a_{\bar{n}\uparrow}^\dagger, \tag{45}$$

where

$$u_{n,\vec{k}} = \int \phi_{\vec{k}}^*(\mathbf{r}) u_n(\mathbf{r}) d\mathbf{r},$$
$$v_{n,\vec{k}} = \int \phi_{\vec{k}}^*(\mathbf{r}) v_n^*(\mathbf{r}) d\mathbf{r}. \tag{46}$$

Note that we pair $\Phi_n(\mathbf{r})[= \frac{1}{U_n} u_n(\mathbf{r})] \uparrow$ and $\Phi_{\bar{n}}(\mathbf{r})[= -\frac{1}{V_n} v_n^*(\mathbf{r})] \downarrow$ (instead of $\phi_{\vec{k}}(\mathbf{r}) \uparrow$ and $\phi_{-\vec{k}}(\mathbf{r}) \downarrow$) by the unitary transformation (11). The generated vacuum state is

$$\tilde{\phi}_{BdG} = \prod_n (U_n + V_n a_{n\uparrow}^\dagger a_{\bar{n}\downarrow}^\dagger)|0>, \tag{47}$$

instead of the BCS ground state

$$\tilde{\phi}_{BCS} = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} a_{\vec{k}\uparrow}^\dagger a_{-\vec{k}\downarrow}^\dagger)|0> . \tag{48}$$

Therefore a pairing constraint is necessary for the unitary transformation (11) to generate the BCS ground state; that is, both $u_n(\mathbf{r})$ and $v_n(\mathbf{r})$ should be proportional to the normal state wavefunction $\phi_{\vec{k}}(\mathbf{r})$ in order to pair $\vec{k}\uparrow$ and $-\vec{k}\downarrow$. For the current-carrying state, we can pair $\vec{k}+\vec{q}\uparrow$ and $-\vec{k}+\vec{q}\downarrow$. Then, $u_n(\mathbf{r}) = U_{\vec{k}} e^{i(\vec{k}+\vec{q})\cdot\mathbf{r}}$ and $v_n^*(\mathbf{r}) = V_{\vec{k}} e^{i(-\vec{k}+\vec{q})\cdot\mathbf{r}}$.

1.6. Relation between pair potential and gap parameter

From Y.-J. Kim, "Pairing constraint on the real space formalism of the theory of superconductivity".

For a homogeneous system, it was shown

$$\Delta(\mathbf{r}-\mathbf{r}') = \int d\vec{k} e^{i\vec{k}\cdot(\mathbf{r}-\mathbf{r}')}\Delta_{\vec{k}}. \tag{49}$$

But this relation is not exact because of the BCS retardation cutoff.

Correct relation may be obtained only after incorporating the pairing constraint into the self-consistency equation. It is given

$$\Delta(\mathbf{r}-\mathbf{r}') = V\sum_{\vec{k}} \frac{\Delta_{\vec{k}}}{2E_{\vec{k}}} tanh \frac{E_{\vec{k}}}{2T} \phi_{\vec{k}}(\mathbf{r})\phi_{-\vec{k}}(\mathbf{r}'). \tag{50}$$

Comparing Eq. (50) with the BCS gap equation, we also find

$$\Delta_{\vec{k}} = \int \phi^*_{\vec{k}}(\mathbf{r})\phi^*_{-\vec{k}}(\mathbf{r})\Delta(\mathbf{r})d\mathbf{r}. \tag{51}$$

In the presence of impurities, one finds that

$$\Delta(\mathbf{r}) = V\sum_{n} \frac{\Delta_n}{2E_n} tanh \frac{E_n}{2T} \psi_n(\mathbf{r})\psi_{\bar{n}}(\mathbf{r}), \tag{52}$$

and

$$\Delta_n = \int \psi^*_n(\mathbf{r})\psi^*_{\bar{n}}(\mathbf{r})\Delta(\mathbf{r})d\mathbf{r}. \tag{53}$$

Eq. (53) was obtained first by M. Ma and P. A. Lee.

1.7. Problem in the microscopic derivation of the Ginzburg-Landau theory

From Y.-J. Kim, "Pairing constraint on the real space formalism of the theory of superconductivity".

Gor'kov's microscopic derivation of the Ginzburg-Landau theory is not valid. [L. P. Gor'kov, Sov. Phys. JETP, **9**, 1364 (1959).] The problem is in using the self-consistency equation which violates the physical constraint of the Anomalous Green's function. Since the local free energy is not well defined for the Cooper pairs as for the hard-core particles, the gradient term may not be derived microscopically.

From the physical constraint of the Anomalous Green's function,

$$F(\mathbf{r}, \mathbf{r}') = F(\mathbf{r} - \mathbf{r}'), \tag{54}$$

we obtain the revised self-consistency equation,

$$\begin{aligned}
\Delta(\mathbf{r}) &= VT \sum_{\omega} \int \Delta(\mathbf{l}) \{G_{\omega}^{\uparrow}(\mathbf{r}, \mathbf{l}) G_{-\omega}^{\downarrow}(\mathbf{r}, \mathbf{l})\}^{P} d\mathbf{l}, &(55)\\
&\neq \mathrm{Ginzburg-Landau\ equation} &(56)\\
&= \mathrm{BCS\ gap\ equation.} &(57)
\end{aligned}$$

2. Reinvestigation of impure superconductors

2.1. History of the theory of impure superconductors

TABLE I. Theories of impure superconductors

	Ordinary impurity	Magnetic impurity
Anderson	$T_c = T_{co}$	
AG	$T_c = T_{co} - \frac{T_{co}}{\pi\omega_D\tau}(\frac{1}{\lambda} + \frac{1}{2})$	$T_c = T_{co} - \frac{\pi}{4}\frac{1}{\tau_s}$
Suhl and Matthias	$T_c \cong T_{co} - \frac{T_{co}}{\lambda\omega_D\tau}$	$T_c = T_{co} - \frac{\pi}{3.5}\frac{1}{\tau_s}$
Baltensperger		$T_c = T_{co} - \frac{\pi}{4}\frac{1}{\tau_s}$
Kenworthy & ter Haar	$T_c \cong T_{co} - \frac{T_{co}}{\lambda\omega_D\tau}$	
Tsuneto	$T_c = T_{co}$	
KO	$T_c = T_{co} - \frac{T_{co}}{\pi\lambda E_F\tau}$ $-$ Weak Loc. correction (Kim)	$T_c = T_{co} - \frac{0.18\pi}{\lambda\tau_s}$

• It is ironic that Kenwhorthy and ter Haar [Phys. Rev, **123**, 1181 (1961)] said that Abrikosov and Gor'kov's (AG) theory of impure superconductors is wrong because of the absence of the correction term $\frac{1}{\omega_D\tau}$. Whereas AG admitted the existence

of the correction term $\frac{1}{\omega_D \tau}$ in their theory.

Strictly speaking, in the electron-electron interaction model under consideration, this conclusion is true only to within terms of order $1/\omega_D \tau \sim 10^{-6} cm/\ell$. (From p. 337 in "Methods of quantum field theory in statistical physics".)

It is also amazing that ter Haar translated the AG's paper[Sov. Phys. JETP, **12**, 1243 (1961)].

2.2. New theory of impure superconductors by Kim and Overhauser

2.2.1 Ordinary impurity case

Recently, Kim and Overhauser (KO)[1] showed the following:

(i) Abrikosov and Gor'kov's (AG) theory[2] of an impure superconductor predicts a large decrease of T_c, proportional to $1/\omega_D \tau$. ω_D denotes the Debye frequnecy and τ is the scattering time, respectively.

$\Longrightarrow$ The existence of the above correction term was confirmed by Abrikosov, Gor'kov and Dzyaloshinskii,[4] and was also shown by other workers.[5–7] The correction term is related with the change of electron density of states due to the impurity scattering. However, the correct value was shown to be $1/E_F \tau$.[1] Here E_F denotes the Fermi energy.

(ii) Anderson's theorem[3] is valid only to the first power in the impurity concentration. For strongly localized states, the phonon-mediated interaction is exponentially small.

$\Longrightarrow$ It is, then, expected that weak localization correction terms occur in the phonon-mediated interaction.

Table I. Mean free path and the phonon-mediated interaction in dirty, weak localization and strong localization limits. Here ℓ and L are the elastic and inelastic mean free paths and α denotes the inverse localization length.

disorder limit	dirty	weak localization		strong localization
ℓ	$\sim 100\AA$	$\sim 10\AA$		$\sim 1\AA$
$V_{mm'}$	V	$V[1-\frac{2}{\pi k_F \ell} ln(L/\ell)]$	$(2d)$	$\sim exp(-\alpha L)$
		$V[1-\frac{3}{(k_F \ell)^2}(1-\frac{\ell}{L})]$	$(3d)$	
		$V[1-\frac{1}{(\pi k_F a)^2}(\frac{L}{\ell}-1)]$	$(1d)$	

[From Y.-J. Kim, Mod. Phys. Lett. B **10**, 555 (1996)].

2.2.1 Magnetic impurity case

For magnetic impurity effects, Kim and Overhauser (KO)[8] also proposed a BCS type theory with different predictions:

(i) The initial slope of T_c decrease depends on the superconductor and is not the universal constant proposed by Abrikosov and Gor'kov(AG).[9]

$$T_c = T_{co} - \frac{0.18\pi}{\lambda \tau_s}. \tag{58}$$

(ii) The T_c reduction by exchange scattering is partially suppressed by potential scattering when the overall mean free path is smaller than the coherence length. This compensation has been confirmed in several experiments.[10−14]

Note that if we impose a pairing constraint on the self-consistency equation or the AG's calculation, we can find KO's result.[15]

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2.3. Strong-coupling theory of impure superconductors

Recently, Abrikosov and Gor'kov[1] argued that the correction term $1/\omega_D\tau$ in their theory disappears in the Eliashberg equation apart from the corrections of the order $1/E_F\tau$. In fact, this result was first obtained by Tsuneto.[2] As a result, they admittedly showed that Gor'kov formalism is inconsistent with the Eliashberg equation.

At this point, we may need to pause to answer the following deep question: Is there a correspondence rule between strong-coupling and weak-coupling theories of impure superconductors? The answer is yes. It can be shown that the correspondence principle, which relates strong-coupling and weak-coupling theories, works provided that Anderson's pairing condition is satisfied.

2.3.1. Strong-coupling theory with Anderson's pairing

The conventional strong-coupling self-consistency equation is[3]

$$\Delta^*(\omega_n, \mathbf{r})Z(\omega_n) \\ = \gamma^2 T \sum_{n'} \lambda(\omega_n, \omega_{n'}) \int d\mathbf{r}_o G_N^{\uparrow}(-\omega_{n'}, \mathbf{r}_o, \mathbf{r}) G_N^{\downarrow}(\omega_{n'}, \mathbf{r}_o, \mathbf{r}) \Delta^*(\omega_{n'}, \mathbf{r}_o) Z(\omega_{n'}). \quad (59)$$

Equation (59) states physically that the pair potential $\Delta^*(\omega_{n'}, \mathbf{r}_o)$ launches (from the regions near $\mathbf{r}_o$) electron pairs which collaborate to generate a pair potential $\Delta^*(\omega_n, \mathbf{r})$ in the region near $\mathbf{r}$. However, Eq. (59) misses the most important information of Anderson's pairing condition. Whereas it was shown[4] that Anderson's pairing condition is derived from the physical constraint of the Anomalous Green's function, i.e.,

$$\overline{F^+(\omega_n, \mathbf{r}, \mathbf{r}')}^{imp} = \overline{F^+(\omega_n, \mathbf{r} - \mathbf{r}')}^{imp}, \quad (60)$$

$$\overline{\Delta^*(\omega_n, \mathbf{r})}^{imp} = \overline{\Delta^*(\omega_n)}^{imp}. \quad (61)$$

Consequently, the revised self-consistency equation is

$$\Delta^*(\omega_n, \mathbf{r})Z(\omega_n) = \\ \gamma^2 T \sum_{n'} \lambda(\omega_n, \omega_{n'}) \int d\mathbf{r}_o \{G_N^{\uparrow}(-\omega_{n'}, \mathbf{r}_o, \mathbf{r}) G_N^{\downarrow}(\omega_{n'}, \mathbf{r}_o, \mathbf{r})\}^P \Delta^*(\omega_{n'}, \mathbf{r}_o) Z(\omega_{n'}), \quad (62)$$

where P denotes Anderson's pairing constraint.

The importance of Anderson's pairing constraint was already noticed by Ma and Lee.[5] They showed that the gap parameter is given by

$$\Delta^*(\omega_n, m) = \int \psi_m(\mathbf{r}) \psi_m^*(\mathbf{r}) \Delta^*(\omega_n, \mathbf{r}) d\mathbf{r}. \quad (63)$$

Substitution of Eq. (63) into Eq. (62) leads to a strong-coupling gap equation

$$\Delta^*(\omega_n, m)Z(\omega_n) = T \sum_{n'} \lambda(\omega_n, \omega_{n'}) \sum_{m'} V_{mm'} \frac{\Delta^*(\omega_{n'}, m')Z(\omega_{n'})}{[\omega_{n'} Z(\omega_{n'})]^2 + \epsilon_{m'}^2}, \quad (64)$$

where

$$V_{mm'} = \gamma^2 \int |\psi_m(\mathbf{r})|^2 |\psi_{m'}(\mathbf{r})|^2 d\mathbf{r}. \quad (65)$$

2.3.2. Comparison with Tsuneto's theory

Tsuneto[2] obtained the gap equation

$$\Sigma_2(\omega) = \frac{i}{(2\pi)^3 p_o} \int dq \int d\epsilon \int d\omega' \frac{qD(q, \omega - \omega')\eta(\omega')\Sigma_2(\omega')}{\epsilon^2 - \eta^2(\omega')\omega'^2}, \quad (66)$$

where $\eta = 1 + \frac{1}{2\tau|\omega|}$, and τ is the collision time.

Comparing Eqs. (64) and (66), we find that Tsuneto's result misses the most important factor $V_{mm'}$, which gives the change of the phonon-mediated interaction due to impurities. This factor is exponentially small for the localized states. In the weak localization limit, it was shown that[6]

$$V^{3d}_{mm'} \cong -V[1 - \frac{1}{(k_F\ell)^2}(1 - \frac{\ell}{L})],$$
$$V^{2d}_{mm'} \cong -V[1 - \frac{2}{\pi k_F\ell} ln(L/\ell)],$$
$$V^{1d}_{mm'} \cong -V[1 - \frac{1}{(\pi k_F a)^2}(L/\ell - 1)], \tag{67}$$

where a is the radius of the wire.

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2.4. Compensation of magnetic impurity effect by radiation damage or ordinary impurity

Compensation of the reduction of T_c caused by magnetic impurities has been observed as a consequence of radiation damage or ordinary impurity. The recent theory by Kim and Overhauser (KO) gives a good fitting to the experimental data.

Note also that Gor'kov's formalism with the pairing constraint derived from the Anomalous Green's function leads to KO theory.

- Compensation by radiation damage

Fig. 1. Superconducting transition temperature T_c of In (open symbols) and

In-Mn (closed symbols) vs Ar fluence. Data are due to Hofmann, Bauriedl, and Ziemann, Z. Phys. B **46**, 117 (1982). $1/\tau_s$ was adjusted in the theoretical curve (lower curve) so that $T_{co} = 1.15K$ without irradiation. [From Park, Lee, and Kim, preprint].

• Compensation by ordinary impurity

Fig. 2 Compensation of T_c in In-Mn-Pb vs Pb concentration for 5-ppm Mn. Data are due to Merriam, Liu, and Seraphim, Phys. Rev. **136**, A17 (1964). [From, Kim and Overhauser, Phys. Rev. B **49**, 15779 (1994)].

2.5. Localization and Superconductivity

For thin films, the empirical formula is given[1]

$$\frac{T_{co} - T_c}{T_{co}} \propto \frac{1}{k_F \ell} \propto R_{\Box}, \tag{68}$$

where T_{co} is the unperturbed value of T_c and $R_{\Box}$ is the sheet resistance. On the other hand, bulk materials show[2,3]

$$\frac{T_{co} - T_c}{T_{co}} \propto \frac{1}{(k_F \ell)^2}. \tag{69}$$

Notice that these results are obtained if we substitute the matrix elements in Table I into the (strong-coupling or weak-coupling) gap equation.

Table I. Mean free path and phonon-mediated interaction in dirty, weak localization and strong localization limits. Here ℓ and L are the elastic and inelastic mean free paths and α denotes the inverse localization length.

disorder limit	dirty	weak localization		strong localization
ℓ	$\sim 100 \mathring{A}$	$\sim 10 \mathring{A}$		$\sim 1 \mathring{A}$
$V_{mm'}$	V	$V[1 - \frac{2}{\pi k_F \ell} ln(L/\ell)]$	$(2d)$	$\sim exp(-\alpha L)$
		$V[1 - \frac{3}{(k_F \ell)^2}(1 - \frac{\ell}{L})]$	$(3d)$	
		$V[1 - \frac{1}{(\pi k_F a)^2}(\frac{L}{\ell} - 1)]$	$(1d)$	

[From Y.-J. Kim, Mod. Phys. Lett. B **10**, 555 (1996)].

Notice that these results are obtained if we substitute the matrix elements in Table I into the (strong-coupling or weak-coupling) gap equation.

• 3 dimension

Fig. 1 Calculated T_c versus resistivity ρ for 3-dimensional Nb_3Ge (dotted line) and V_3Si (solid line). Experimental data are from J. M. Rowell and R. C. Dynes, unpublished. [From Kim and Chang, preprint (1997)].

• **2 dimension**

Fig. 1 Calculated T_c versus sheet resistance $R_\Box$ for a-MoGe (solid line) and Mo-C (dotted line) thin films. Experimental data for a-MoGe and Mo-C are from ref. 4 and 5, respectively. [From Kim and Chang, preprint (1997)].

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3. Study of high Tc superconductors

3.1. Impurity scattering in a d-wave superconductor

For a d-wave superconductor, the pairing interaction $V_{\vec{k},\vec{k}'}$ for the plane states is taken to be[1]

$$V_{\vec{k},\vec{k}'} = \int e^{i(\vec{k}-\vec{k}')\cdot\vec{r}} V(\mathbf{r}) d^3r = -5V_2\ \frac{1}{2}[3(\hat{k}\cdot\hat{k}')^2 - 1], \tag{70}$$

where $\hat{k}$ is the unit vector parallel to $\vec{k}$. Substituting Eq. (70) into the BCS gap equation, one finds

$$\Delta_{\vec{k}} = 5V_2 \sum_{\vec{k}'} \frac{1}{2}[3(\hat{k}\cdot\hat{k}')^2 - 1]\frac{\Delta_{\vec{k}'}}{2E_{\vec{k}'}} tanh \frac{E_{\vec{k}'}}{2T}, \tag{71}$$

where

$$E_{\vec{k}'} = \sqrt{\epsilon^2_{\vec{k}'} + |\Delta_{\vec{k}'}|^2}, \tag{72}$$

and $\epsilon_{\vec{k}}$ is the electron energy. Among the possible solutions, we consider

$$\Delta_{\vec{k}} = \Delta_o(\hat{k}^2_x - \hat{k}^2_y). \tag{73}$$

This solution has the same symmetry property as $d_{x^2-y^2} = \Delta_o(cosk_x - cosk_y)$ which is believed to describe the gap structure of the cuprate high T_c superconductors.

Non-Magnetic Impurity Effect

In the presence of impurities, the scattered states ψ_n may be expanded in terms of plane waves, such as[2]

$$\psi_n = \sum_{\vec{k}} e^{i\vec{k}\cdot\mathbf{r}} < \vec{k}|n > . \tag{74}$$

Now the pairing interaction $V_{nn'}$ between scattered basis pairs $(\psi_n, \psi_{\bar{n}})$ and $(\psi_{n'}, \psi_{\bar{n}'})$ is given by

$$V_{nn'} = \int\int d\mathbf{r}_1 d\mathbf{r}_2 \psi^*_{n'}(\mathbf{r}_1)\psi^*_{\bar{n}'}(\mathbf{r}_2)V(|\mathbf{r}_1 - \mathbf{r}_2|)\psi_{\bar{n}}(\mathbf{r}_2)\psi_n(\mathbf{r}_1). \tag{75}$$

Here $\psi_{\bar{n}}$ denotes the time-reversed state of ψ_n. From Eqs. (70), (74) and (75) we can calculate $V_{nn'}$.

The pairing interaction is reduced:

$$V_{\vec{k},\vec{k}'} = -5V_2 \frac{1}{2}[3(\hat{k}\cdot\hat{k}')^2 - 1][1 + \frac{3.5\xi_o}{4\ell}]^{-2}. \tag{76}$$

Notice that in dilute limit the reduction is proportional to the ratio of the average correlation length to the mean free path, ξ_o/ℓ and the pairing interaction decreases linearly with the impurity concentration. The T_c equation is now,

$$T_c = 1.13\epsilon_c e^{-1/N_o V_2[1+\frac{3.5\xi_o}{4\ell}]^{-2}}. \tag{77}$$

Figure 1 shows T_c versus $1/\tau$ for $T_{co} = 40K$ and 80K respectively. T_{co} denotes the transition temperature without impurities. We used $\epsilon_c = 500K$. For a metal with $v_F = 2 \times 10^7 cm/sec$, the superconductivity is completely suppressed when the mean free paths are about $1000\AA$ and $350\AA$ for $T_{co} = 40K$ and 80K, respectively.

Fig. 1 Variation of T_c with impurity concentration (measured in terms of the scattering rate, $\frac{1}{\tau}$) for $T_{co} = 40K$ and $80K$, respectively. The cutoff energy ϵ_c is $500K$.

Discussions

In high T_c superconductors, the impurity doping and ion-beam induced damage[3] suppress strongly T_c. But the T_c reduction is not fast enough to be explained by this study. The experimental data show that T_c reduction is closely related with the proximity to a metal-insulator transition caused by the impurity doping and the ion-beam-induced damage.[3–5] It seems that the local fluctuations of the gap parameter near the impurities may decrease the effect of impurities in the dirty limit. [From Park, Lee, and Kim, Mod. Phys. Lett. B **11**, 719 (1997)].

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3.2. Impurity doping effect in high Tc superconductors

It has been observed that impurity doping and/or ion-beam-induced damage in high T_c superconductors cause a metal-insulator transition and thereby suppress the critical temperature. Based on my theory of weak localization effect on superconductivity, I examined the variation of T_c with increasing of impurity concentration (x) in $\mathrm{La_{1.85}Sr_{0.15}Cu_{1-x}A_xO_4}$ systems, where A = Fe, Co, Ni, Zn, or Ga. The doping impurity decreases the pair-scattering matrix elements, such as $V_{nn'} = -V[1-\frac{2}{\pi k_F \ell} ln(L/\ell)]$, where L and ℓ are the inelastic and elastic mean free paths, respectively. Using the mean free path ℓ determined from resistivity data, we find good agreements between

calculated values for T_c and experimental data except Ni-doped case. [See Kim and Chang, preprint (1997)].

Fig. 1 Variation of T_c with dopant concentration for $\mathrm{La_{1.85}Sr_{0.15}Cu_{1-x}Ga_xO_4}$. Experimental data are from [Xiao, Cieplak, Xiao, and Chien, PR B **42**, 8752 (1990)].

Fig. 2 Variation of T_c with dopant concentration for $\mathrm{La_{1.85}Sr_{0.15}Cu_{1-x}Fe_xO_4}$.

3.3. On the mechanism of high Tc superconductors

In Ni-doped case, T_c suppression is much slower than expected. This implys that Ni may enhance the pairing interaction in high T_c superconductors such as LSCO, YBCO(123), and YBCO(124). It is also interesting that Ni impurity in YBCO(123) acts as an unpaired spin of $S = \frac{1}{2}$ rather than $S = 1$ expected for Ni^{2+}.[17] Further microscopic study on Ni-doped samples may give a clue to understanding the mechanism of high T_c superconductors. In particular, I am calculating the electronic structure of Ni-doped samples by the exact diagonalization of finite-size clusters.

Fig. 1 Variation of T_c with Ni concentration for $La_{1.85}Sr_{0.15}Cu_{1-x}Ni_xO_4$.

3.4. Search for new high Tc superconductors

La_2NiO_4 is a Mott-Hubbard insulator consisting of antiferromagnetic NiO_2 planes as La_2CuO_4.[1] However, $La_{2-x}Sr_xNiO_4$ remains nonmetallic until $x \geq 0.8$ because the holes doped into the NiO_2 planes tend to order themselves in periodically spaced stripes.[2–4] There is also evidence for related stripe correlations in hole-doped La_2CuO_4.[5–7]

The stripe order seems to localize the holes. So we need to study how to induce metallic phase and concomitant superconductivity in $La_{2-x}Sr_xNiO_4$. One possibility is to dope a large amount of Cu into the NiO_2 planes in order to increase the mobility of the holes. *If Cu and Ni order in the planes, T_c may be very high.* Another possibility is to substitute O by N, F, or other elements, in order to increase the hybridization between 3d and 2p orbitals. Then, the system may become metallic and superconducting.

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4. New formalism for inhomogeneous superconductors

This chapter is still in progress.

4.1. Generalization of the BCS theory

We may distinguish the impurity problem and the junction or the vortex problem. When we consider the impurity problem using Anderson's approach, the local fluctuation of the gap parameter is disregarded. However, since the latter two problems are related to the macroscopic or mesoscopic inhomogeneity, we should allow the gap parameter to vary as a function of the position. Accordingly, we need to generalize the BCS theory to tackle the macroscopically or mesoscopically inhomogeneous systems.

A key idea is to introduce the position-dependent Cooper-pair size into a BCS type wavefunction.

4.2. Generalization of the Ginzburg-Landau theory

The above study may lead to an extention of the Ginzburg-Landau theory on the macroscopic level. Because the Ginzburg-Landau theory appeared before the pairing theory of BCS, it lacks the effect of the Cooper pair-size. The Ginzburg-Landau coherence length ξ_{GL} is the characteristic length for variation of the order parameter. This ξ_{GL} is not the same length as the BCS coherence length ξ_o. Note that the local free energy density is not well defined in superconductors because of the Cooper-pair size.

It is very important to incorporate the information of the pairing correlation into the traditional Ginzburg-Landau theory.

5. Future directions

5.1. Impure superconductors

Ordinary impurity case

Anderson's approach can be used to restudy the following the topics:

1. Thermodynamic properties,
2. Electrodynamics,
3. Coherence effects,
4. Response functions,
5. Strong-coupling theory using the realistic phonon model.

Superconducting behavior very near an impurity may not be understood by Anderson's approach. We need a more general formalism which can take into account

the variation of the gap parameter near the impurity.

Magnetic impurity case

KO theory may used to restudy the following the topics:

1. Thermodynamic properties,
2. Electrodynamics,
3. Coherence effects,
4. Response functions.

It is clear that compensation of the magnetic impurity effect by radiation damage or ordinary impurity should be subjected to further experimental study.

5.2. Localization and Superconductivity

In the weak localization limit, I showed that

$$V^{3d}_{mm'} \cong -V[1 - \frac{1}{(k_F\ell)^2}(1 - \frac{\ell}{L})],$$
$$V^{2d}_{mm'} \cong -V[1 - \frac{2}{\pi k_F\ell} ln(L/\ell)],$$
$$V^{1d}_{mm'} \cong -V[1 - \frac{1}{(\pi k_F a)^2}(L/\ell - 1)],$$

where a is the radius of the wire.

Using the above matrix elements, we may study the following problems:

1. Thermodynamic properties,
2. Electrodynamics,
3. Coherence effects,
4. Response functions,
5. Effect of spin-orbit scattering
6. Strong-coupling theory.

The so-called superconductor-insulator transition may be also understood. It seems that the superconductor-insulator transition is not a sharp phase transition but a crossover phenomena from quasi-wto dimensional to two-dimensional. Note

that the critical sheet resistance for the suppression of superconductivity in this films is not a universal constant, but a sample-dependent quantity.

5.3. Proximity Effect

Note that Gor'kov formalism with a pairing constraint leads to the revised self-consistency equation which is nothing but another form of the BCS gap equation.[1] Accordingly, both the revised gap equation and the BCS gap equation are useless in describing the proximity effect. It is understandable that proximity effect is a long-standing unsolved problem. We need a new formalism to determine how fast the Cooper-pair size is changing in the normal region.

Note that the proximity effect in mesoscopic superconducting junctions shows anomalous behaviors.[2,3]

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5.4. Andreev reflection

Recently, it has been realized that the distinction between the proximity effect and Andreev reflection is artificial.[1] Consequently, the conventional theory of Andreev reflection is not complete. A unfied theory of the proximity effect and Andreev reflection is required.

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5.5. Josephson Effect

Supercurrents are found in SIS, SNS, and S-semi-S structures. The thickness dependence of the supercurrents are not well understood. In particular, the recent

mesoscopic S-semi-S junctions show anomalous behavior of the supercurrents.[1] If a unified theory of the proximity effect and Andreev reflection is constructed, the theory may shed light on this problem.

Note also the sign problem in the pair-quasi-particle interference term.[2]

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5.6. Magnetic field effect

KO[1] theory may be useful in determining the magnetic field effect on superconductors. Notice the discrepancies in existing thoeries.

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The reentrant superconducting state in very high magnetic field may be an artifact caused by the pairing problem in Gor'kov's formalism.

5.7. Type II superconductors

The magnetic behavior of Type II superconductors may need to be reexamined. H_{c1}, H_{c2}, vortex, and flux pinning, creep, and flow are aprticularly interesting. We need a new formalism to tackle these problems.

5.8. Vortex problem

Recent STM experiments show that the microscopic vortex structure is very complicated. The conventional Green's function theory is not applicable to this problem. We had better solve one vortex problem using a new microscopic formalism.

5.9. Mesoscopic superconductivity

Recently, much attention has been paid to this topic.[1,2,3] It is clear that our understanding of inhomogeneous superconductors is in its infancy.

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5.10. Non-equilibrium superconductivity

Non-equilibrium superconductivity is similar to inhomogeneous superconductors in that the Cooper-pair size may vary as a function of the position.

5.11. Granular superconductors

Granular superconductors are related to the macroscopic or mesoscopic inhomogeneity.

5.12. High Tc superconductors

Since high Tc superconductors are strongly correlated, both normal and superconducting properties are significantly influenced by the correlation effect. We need to know how to take into account properly this effect. For an example, the experiments clearly show that impurity potential is strongly renormalized by correlation.